

ON THE OF MATHEMATICAL THEORIES II

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This article is the concluding text about Putnam's philosophical position in the philosophy of mathematics. This position was given of the beginning of the article »On the Nature of Mathematical Theories I« and now we have shown its its consequences.

Let us look again at what we have already said about Putnam's alternative modal picture of mathematics. It is our intention to complete this picture of mathematics with the results from Putnam's project of refusing any need for distinguishing special mathematical entities. So far we have seen that Putnam introduces his modal picture through rejecting any ontology which would speak about existence of any kind of such entities. In his modal picture a mathematician speaks about what is mathematically possible or impossible. This mathematical possibility rests on the notion of simple consistency whose validity is secured by means of uncontradictory ω — sequence. But, the truth is decided by means of empirical content. Thus, mathematical potentiality is equivalent to one part of the modal-logic mathematical picture, and its actuality (empirical realisation, that is, the knowledge of truth) to its other part. The relativity of the autonomy of the first part of the modal-logic picture of mathematics can be seen by the fact that mathematical special objects can be replaced by objects from any domain, even by physical objects. The relativisation of this part of the modal-logic picture of mathematics, together with the epistemological aspect of connecting the truth of mathematical propositions with physical experience (which at the same time means knowledge of that experience), enables this modal-logic picture of mathematics to be brought in to accordance with the project of identification (mathematics/physics). The position of nonexistence of a special abstract domain of mathematics enables the elimination of possible platonism and establishes the identification with empirical sciences as a realistic alternative. It follows that for mathematics it is completely irrelevant whether we ask the question about the nature of special mathematical entities or not.

Then the equivalences $P \equiv P^*$ discussed before would be forthcoming as theorems. It is these equivalences that underline the

logician account of the application of mathematics; how exactly the numbers are defined, or whether they are taken as primitive is immaterial as long as these equivalences can be derived.«¹

The mode of existence of special mathematical structures becomes in this way completely different from mathematical (platonistic) object — picture. Namely, this relatively autonomous part of mathematics gives the complete modal meaning to the notion of the existence of mathematical structures, owing to its axioms taken from applied mathematics. This modal meaning is the concept of possibility:

»Thus applied mathematics does *not* presuppose that models for our mathematical axiom-sets ('standard' or 'non-standard') actually do exist, but, only that they *could* exist.«²

To confirm that Putnam holds this modal-logic picture of mathematics in his later works we can use the following citation:

»The notion of possibility does not have to be taken as a *primitive* notion in science. We can, of course, define a structure to be *possible* (mathematically speaking) just in case a model exists for a certain theory, where the notion of a model is the standard set theoretic one. That is to say, we can take the existence of sets as basic and treat possibility as a derived notion. What is often overlooked is that we can perfectly well go in the reverse direction: we can treat the notion of possibility as basic and the notion of set existence as the derived one. Sets, to parody John Stuart Mill, are permanent possibilities of selection.«³

But as he must admit the autonomy of mathematics (in the last consequence for historical reasons) Putnam must retain a special mathematical method which, although it accords with the methods of empirical sciences, still keeps the speciality of particular mathematical structures. But, lest the concept of mathematical structure should take him back to Platonism, Putnam includes this concept into his modal-logic picture of mathematics. So, the combination of: mathematical possibility (possible mathematical structures), binding of the chosen axioms to applied mathematics, mathematical necessity (validity in all meaningful models), makes possible for realism the modal mathematical picture, that is, the project mathematics/physics. It is clear that he derives the validity of all meaningful models from the mathematical applicability and so we discover the reason why, according to Putnam, it is not

¹ Putnam H. *Mathematics Matter and Method* Cambridge University Press, Cambridge 1975. p. 38.

² *ibidem*, p. 33.

³ *ibidem* p. 33.

necessary to take various mathematical »isms« seriously. In this way Putnam establishes the necessity of unification of various philosophical attitudes toward mathematics, that is, the unification of various mathematical theories:

»For our intuitive conviction that certain kinds of infinite structures *could* exist plays an essential role in the *application* of mathematics. It is a part, and an important part, of the *total* mathematical picture that certain sets of axioms are taken to describe *presumably possible* structures. It is only *such* sets of axioms that are used in *applied* mathematics.«⁴

Inside of the frame of his endeavor for a mathematics without foundations Putnam confirms that the avoidance of the mathematical object-picture does not mean establishing new kinds of special mathematical objects:

»Introducing the modal connectives is not introducing new kinds of objects, but rather extending the kinds of things we can say about ordinary objects and sorts of objects.«⁵

At first sight it might seem as if Putnam's modal-logic picture does not necessarily result with the unification of mathematical theories or with reduction, that is, identification of mathematics with the empirical sciences. Because: »... not only that the objects of pure mathematics are conditioned; they are, in a sense, simply abstract possibilities.« But, as Putnam does not permit a priori propositions in mathematics, because »... mathematical knowledge resembles empirical knowledge«, it is clear that realism in mathematics can be supported by this »resemblance« to complete identification.

»... that the physicist who states a law of nature with the aid of a mathematical formula is abstracting a real feature of a real material world, even if he has to speak of numbers, vectors, tensors, state-functions, or whatever to make the abstraction.«⁶

Now, it is quite clear that Putnam has to reject mathematical apriorism. The result of such a position can be a certain insecurity in connection with Putnam's philosophical position about the question of mathematical method. For example:

»From classical mechanics through quantum mechanics and general theory relativity, what the physicist does is to provide mathe-

⁴ ibidem p. 71.

⁵ ibidem p. 41.

⁶ ibidem p. 58--59.

mathematical devices for representing all the *possible* — not just the physically possible, but the mathematically possible — configurations of a system.⁷

Isn't there a certain ambiguity about the status of mathematical

method? Namely, the distinguishing of mathematically possible forms of systems from physically possible forms of systems surely leads to the autonomy of mathematical method. On the other hand, the hypothetical nature of mathematical propositions, since they are only possible suppositions of a system, leads to an antirealist understanding of mathematical method (the so called »if — thenism«). Now, it is not even important whether it is an aprioristic or a nonaprioristic position in the philosophy of mathematics, because »if — thenism« is possible in the case of choosing axioms from applied mathematics.

On the other hand, there is the identification of mathematics and physics through the account of the method as an mathematical/physical invention of describing physical systems.

If we compare citation number 7 with citation number 6 then the »rigidity« of Putnam's epistemological position in relation to mathematical method, various mathematical theories and the autonomy of mathematics does not leave any doubt that his position is ambiguous. This position was given at the beginning of this article and now we have shown its consequences.

Mirko Jakić: O PRIRODI MATEMATIČKIH TEORIJA II

Sažetak

Članak je završni nastavak prethodnog pod istim naslovom. Putnamov stav u filozofiji matematike je doveden do ontološko logičkih posljedica i do tvrdnje o viseznačnosti ideje eksternalnog realizma u njegovoj filozofiji.

⁷ ibidem p. 60.

⁸ ibidem p. 71.