

## ON KITCHER'S OBJECTION TO THE APRIORIST PROGRAM IN MATHEMATICS

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UDK/UDC:510  
Izvorni znanstveni članak  
*Original scientific paper*

Primljeno : 1990-12-28  
*Received*

This article deals with Kitcher's objection against the concept of *a priori* knowledge which is achieved in mathematics. The analysis of Kitcher's argumentation shows that his objection is fallible.

Kitcher's objection to the aprioristic program in mathematics is based on the denial of the possibility of epistemological thesis that mathematical theories are forms of the knowledge of *a priori* character.<sup>1</sup>

At this point I will try to state Kitcher's description of the concept of a priori. Then, basic assumption of his psychological warrant of a priori knowledge and his criticism of aprioristic program in mathematics.

Kitcher's description of the concept of a priori knowledge is the following:

"A priori" is an epistemological predicate.

A priori "knowledge absolutely independent of all experience" it implies a possibility of untransformed true belief in all possible worlds (regardless of properties of possible particular empirical experience). If, as person's experience, we count the stream of her sensory encounters with the world, we include external experience with the external causes, external to the body. In this case, the experience obtained by the external causes is external experience. Internal a priori

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<sup>1</sup> I refer to Kitcher, P. (1980) and Kitcher (1984)

experience is absolutely independent of any kind of particular external experience. Absolutely independent a priori knowledge involves the independence from the total experience of the knower. There is a necessary a priori belief and a contingent one. The difference between the concept of knowledge and the concept of belief is the following: knowledge is an item of warranted true belief. Consequently, the true belief, because of the epistemological inadequacy, does not represent knowledge. Only true belief warranted with the reliable process represents an item of knowledge.

A priori knowledge is true belief warranted with the process which is its a priori warrant. The same process always produces the same type of belief.

Because it is possible to have empirical knowledge about the statements attainable a priori, we must distinguish such knowledge from the actual a priori knowledge.

The reliable process which produces warranted belief as an item of knowledge is particular internal psychological process - the process of the particular type which always could produce the same belief. In the case of a priori belief the psychological process ought to be such process which could always produce reliable a priori warrant belief as a priori knowledge. Because of that we can call Kitcher's proposal as psychological interpretation of the epistemological status of the kinds of knowledge.

The basic points of his analysis of a priori knowledge are the following:

(1)  $X$  knows a priori that  $p$  iff  $X$  knows that  $p$  and  $X$ 's belief that  $p$  was produced by a process which is an *a priori* warrant for it.

(2)  $P$  is a priori warrant for  $X$ 's belief that  $p$  iff  $P$  is process such that, given any life  $g$ , sufficient  $X$  for  $p$ , then:

- (a) some process of the same type were to produce in  $X$  a belief that  $p$ ,
- (b) if a process of the type were to produce in  $X$  a belief that  $p$  then it would warrant  $X$  in believing that  $p$ ,
- (c) if process of the same type were to produce in  $X$  a belief that  $p$ , then  $p$ .

Therefore, psychological process which should produce a priori belief, and get a priori warrant it as a priori knowledge, is always of the same type (in the sense of nonvariability in different counterfactual situations). Respectively, if a person

demands that some particular belief is an item of a priori knowledge, she must determine that part of the causal descent of the belief which consists of the state and the process which are internal to the believer in an identity type of conditions which fit some set of principles of classification standardly used in differentiation of the process which form belief.

If (2) is satisfied, then the process is of the same kind with no regard of the difference of empirical experience.

A priori knowledge must be ultrareliable, because : if some body is able to ignore empirical information from the world he inhabits, then that can happen only because he has on disposition a method which leads to belief as true warranted belief, This is what (2c) expresses.

Examples of the a priori belief ought to be statements as such: "I am existing", "I have some beliefs", "There are thoughts". Hence, a statement of ourselves and their logical consequences.

Kitcher criticises "logical positivism" as apychological epistemological theory rival to his own, that is - apychological attempt to warrant the a priori knowledge. Because of harmony between the "logical positivism" and apriorism in mathematics (encompassing conceptualism, constructivism, realism) it naturally follows, in his opinion, that criticism of the "logical positivism" simultaneously means a criticism of mathematical apriorism.

According to Kitcher, the basic assumptions of mathematical apriorism are the following:

- (3) There is a class of statements A and a class of rules of inference R such that:
- (a) each member of A is a basic a priori statements,
  - (b) each member of R is an apriority-preserving rule,
  - (c) each statement of standard mathematics occurs as the last member of a squence, all of its members either belong to A or come from previous members in accordance with some rule in R.

The basic Kitcher's objection is inferred from the fact that there is significant uncertainty in the case of deciding the truth value of statements which are obtained by extremely long mathematical inferences , in the procedure of the theorem proof. Reasons for this loss of certainty is inability to concentrate on every single step of the inference and inability to keep in mind all the relevant elements of inference.

Besides that, "logical positivism", as the core of mathematical apriorism takes that all mathematical statements are analytical. Consequently, that means that we have preanalytical knowledge about the basic mathematical statements, and, with mathematical analysis, do not enter a process of epistemic kind.

Kitcher offers us two basic reasons for rejection of mathematical apriorism. First, there are no basic a priori apsihological warranted mathematical statements. Secondly, because of the significant uncertainty of mathematical statements obtained with the help of long proofs, this is the reason why we could not call it a priori knowledge. Further, in case of adoption of apriorism, some variant of (3) holds:

For let  $S$  be any true standard mathematical statement. By (3c) there is a sequence of sentences with its members by one the rules in  $R$ .

We can show by induction, using (3a) and (3b), that every statement in the sequence is knowable a priori. Consequently,  $S$  is knowable a priori and every truth of standard mathematics is knowable a priori. This argument Kitcher calls inductive argument.

Furthermore, in the case of rejection of apriorism (namely rejection of all variants of (3)), the following holds:

There are true standard mathematical statements  $S$  such that the shortest proof of  $S$  would require even the most talented human mathematicians to spend months in concentrated effort to follow it. Because of the rational worry about the value of such statements we can infer inevitable uncertainty and non-existence of a priori knowledge.

Faced with this pair of contradictory suggestions, according to Kitcher, we must choose among three possible options:

- (i) we can accept the inductive argument and the point about long proofs, concluding that no versions of (3) can be correct,
- (ii) we can accept the inductive argument and reject the point about long proofs, thereby concluding that (3) suffices to establish mathematical apriorism,
- (iii) we can reject the inductive argument, concluding that (3) does not suffice to establish apriorism.

Since, it is obvious that the central Kitcher's proof against the apriorist program in mathematics is based on the nonagreement between the demand for ultrareliability of the a priori knowledge and the factual uncertainty connected with the problem of the long proofs. It is possible, according to Kitcher to get two "positivistic/aprioristic" objections on his conclusion that demand for ultrareliability of apriority is of the central value. As follows:

(I) Uncertainty is a contingent element of some situations which belongs to the particularity of method, namely, to the nature of proof we follow.

## (II) A priori knowledge is compatible with rational uncertainty.

Kitcher's rejection of the objection (I) is as follows:

Rational uncertainty is unavoidable when we are in the process of proving a theorem. Formalisation, in the case of the increase in the length of the sequence of proof, can only exacerbate our rational worry that mathematician lost his concentration or that he remembered incorrectly some previous result. It is not possible to make rational uncertainty significantly less than it usually is.

A priori knowledge necessarily demands absolute certainty. So, it is necessary to accept (i) as conclusion.

His rejection of the objection (II) is the following:

If we state that a priori knowledge is compatible with the rational uncertainty, we must conflate a priori knowledge with knowledge obtained by following non-empirical process. In that case condition (2b) of the a priori warrant is not satisfied. Hence, (i).

Generally: *No one block the inference from uncertainty to the absence of apriority. Rational uncertainty does not preclude knowledge, but it does rule out a priori knowledge.*

Consequently, we can not have confidence in (3b), namely in the reliability of the rules of logic. The rules of logic are reliable only in case when we focus on a problem in a step-by-step way. But, when we have a problem which demands simultaneous attention to all the inferential steps, the inability of the rules of logic as apriority-preserving transmitters become apparent. So, the unreliability of the rules of inference, *modus ponens* for example, results from its inability as apriority-preserving rule - apriority is incompatible with uncertainty.

For the case of objection to the attitude that it is impossible to diminish the rational uncertainty when we are dealing with the long proofs by computer's assistance, Kitcher answered that we are now dealing with the rational worry about the possibility of technical errors. So, uncertainty is constant.

Here I will quote two of Kitcher's attitudes which are very important for my next criticism.

"The apriorist emphasis on the importance of proofs in mathematics reflects a traditional answer: proofs codify psychological processes which can produce apriori knowledge of the theorem proved. If we are to embed the popular thesis that mathematical knowledge is a priori because it is based on proof in an adequate epistemology, then I submit that this is the answer which we should adopt".<sup>2</sup>

<sup>2</sup> Kitcher, P. (1984), p.37

So, according to Kitcher, it is necessary to accept that every mathematical phenomenological state is simultaneously psychological state. Every psychological state is product of a psychological process. Then, a relation between psychological process and psychological state is relation between cause-producer and effect-product.

"In rejecting mathematical apriorism, I am not committed to any conclusions about the modal status of mathematical truths. It is quite consistent to claim that mathematical truths are necessary, even that we know that they are necessary, and to deny that our mathematical knowledge is a priori.

Although I shall not argue for the necessity of mathematics, I believe that the thesis that mathematical truths are necessary is defensible and that my critique of mathematical apriorism is irrelevant to it. From the perspective of much traditional thinking about 'the a priori', these claims will sound absurd. The notions of apriority and necessity have been so closely yoked in philosophical discussion that 'a priori' and 'necessary' are sometimes used as if they were synonyms. They are not. We have already discovered that 'a priori' and 'necessary' are not even coextensive by finding examples of contingent truths which can be known a priori. What I now want to show is that necessity does not imply apriority".<sup>3</sup>

## II

At this point I will try to present some doubts that Kitcher successfully refuted the possibility of mathematical aprioristic program. But, if the concept of "logical positivism" really implies that it is possible to have mathematical knowledge about basic relations among mathematical axioms in preanalytical sense, namely, that mathematical knowledge exists from the beginning of the process of proving, then my criticism is not from the point of view of "logical positivism".

My attitude is that our intuitive access rests upon the basic mathematical statements and upon the rules of logic. Mathematical analysis means acquiring knowledge about the possible mathematical structures. Contrary to my attitude and according to "logical positivism" in Kitcher's interpretation, mathematical analysis should not be necessary, because all mathematical knowledge should be taken in by an intuitive preanalytical access. Furthermore, I agree with Kitcher that psychological process is the cause of a psychological state. Especially because, in that case the psychological process discovered as the cause of particular mathematical "phenomenal" state can be treated as the central element of the psychological explanation of mathematical "phenomenal" state.

Here are some criticisms of Kitcher.

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<sup>3</sup> Kitcher, P. (1984), p.32

*On ultrareliability of a priori knowledge:*

I will try to show that no property of the concept of a priori knowledge accepted by Kitcher implies the concept of ultrareliability. According to Kitcher, the concept of apriority involves: an epistemic predicate, statements which we can know a priori, knowledge absolutely independent of all empirical knowledge, truth in all possible worlds of empirical experience, experience absolutely internal, the cause of a *a priori* knowledge is particular psychological process which is its a *priori* warrant.

It is obvious that, of the all properties of the concept of a *priori* knowledge, the single candidate which may link to ultrareliability is the property of "being true in all possible worlds of empirical experience". It is clear that this property is derivative from the modal concept of necessity. If the property of "being true in all possible worlds of empirical experience", as derivative of the modal concept of necessity, leads to ultrareliability, we must allow that the concept from which this is derivative is itself derived. Namely, the modal concept of necessity also leads to ultrareliability. If the concept of apriority necessarily implies ultrareliability, it must necessarily imply the modal concept of necessity. According to Kitcher (see second quotation) this is not the case. So, if necessity does not necessarily imply apriority and apriority necessarily implies ultrareliability, then it is impossible to infer ultrareliability of the apriority from necessity.

*On ultrareliability of the psychological process which warrant that true a priori belief is a priori knowledge:*

It is certain that Kitcher agrees to some extent about the possibility of aprioristic understanding of the nature of mathematical proofs under the conditions of appropriate epistemology.

That adequate epistemology is his psychologism (see the first quotation). But in that case, psychological processes which produce a *priori* mathematical knowledge are, according to Kitcher, sufficient for the warrant of beliefs in the sense of acquiring a *priori* knowledge. But logic is the result of processes of the same kind. Why can not we then justify mathematics by logical remedies if we count them as the effects of these very same psychological causes? The answer, says Kitcher, is uncertain. Furthermore, such psychological processes are not discovered. They can be discovered only empirically, (except in the case when it is possible to maintain that psychology is an a *priori* science). With the recognised rational uncertainty of empirical knowledge the result is that the knowledge about

such processes should be uncertain. Then, such processes which should enable *a priori* knowledge imply rational uncertainty. We must ask: What is left of the ultrareliability of the presupposed psychological processes which should warrant *a priori* knowledge?

*On uncertainty of the logical rules of inference:*

According to Kitcher, a nondubious example of *a priori* knowledge is the statement "I am existing" and the statement "There are thoughts" and its logical consequences. Then, inference with *modus ponens* must be an item of *a priori* knowledge. For example:

If I am thinking it is impossible that I do not exist,  
and this is true in all possible worlds regardless of the mode of existence;  
I am thinking.  
So, I am existing

It is clear and Kitcher does not deny this, that the logical rule of inference can be significant part of the *a priori* knowledge.

But, in the case of long proofs logical rule of inference shows their inability as apriority-preserving rule. It is obvious that here we are dealing with the use of logical rule of inference. On one occasion, according to Kitcher, when we use it for particular inference, it is reliable. On another occasion, when we use it in long inferences, (when it is possible to follow simultaneously all steps of inference), it is unreliable.

But, if it was possible to follow, even simultaneously, every step of inference in particular, then the logical rule of inference would necessarily be reliable. Uncertainty of the logical rule of inference is not the property of that rule, but it is property of the way we use it.<sup>4</sup>

Finally, we can conclude that, despite Kitcher's objections, aprioristic program in mathematics still remain as knowledge absolutely independent of any kind of

<sup>4</sup> Kitcher's objection on the possibility of improvement of certainty of decision of the truth value of the statements inferred in many steps of inference by computer assistance is beyond the scope of this article. But disagreement with Kitcher's attitude can be established in two main points: a) It is possible to deny that possible human errors and computer errors are of the same kind. We can improve computers in technical sense, but we can not improve human abilities. b) As contingent technical identification, computer does not change the *a priori* status of mathematical knowledge.

empirical experience, but not as the ultrareliable, nonfallible knowledge. It is quite acceptable that every kind of human knowledge is fallible regardless whether it is *a priori* knowledge or not.

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*Mirko Jakić*: O KITCHEROVU PRIGOVORU APRIORISTIČKOM PROGRAMU U MATEMATICI

S a ž e t a k

Ovaj se članak bavi Kitcherovim prigovorom protiv pojma a priori znanja u matematici.

Analiza Kitcherove argumentacije pokazuje da je njegov prigovor pogrešan.